SIMULATION OF A PANCHROMATIC BAND BY SPECTRAL LINEAR COMBINATION OF MULTISPECTRAL BANDS

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1. INTRODUCTION

The simulation of satellite imagery is a useful tool as an image processing technique. One could mention at least two applications in the remote sensing area. a) The large sizes of remote sensing images impose a severe burden on the communications link between the satellite and the ground. Therefore, the simulation of an image by ground processing could offer the potential of a decrease on the data rate. b) In sensor feasibility studies it is common practice to develop simulation procedures before the actual sensor is built, so that many technical aspects can be predicted in advance.

The main idea of this paper is to present a simulation technique for a sensor band based on linear combination of other bands. This is possible if there is a substantial spectral overlap between the simulated band and the bands that are linearly combined. As an example of this procedure, we present a study of the simulation of a degraded, 20 m resolution SPOT panchromatic band (P), by linear combination of the two multispectral bands (XS1 and XS2) that spectrally overlap the panchromatic band. The use of the third multispectral band (XS3) was discarded because of the negligible spectral overlap between P and XS3.

2. THEORETICAL BACKGROUND

Let \( f(x) \) be the spectral characteristic of the scene, \( g_1(x), i=1,2 \) be the spectral characteristics of the two multispectral bands and \( g_p(x) \) the spectral characteristic of the panchromatic band.

The observed value at each pixel of the multispectral bands can be approximately given by:

\[
\rho_i = \int f(x) g_i(x) \, dx \quad i = 1,2
\]  

Likewise, for the panchromatic band

\[
\rho_p = \int f(x) g_p(x) \, dx
\]  

We want to estimate \( \rho_p \) by \( \rho \), where \( \rho \) would be given by a fictitious band that is a linear combination of the two multispectral bands, i.e.

\[
\rho = \int f(x) g_\rho(x) \, dx
\]  

where \( g_\rho(x) = \beta_1 g_1(x) + \beta_2 g_2(x) \) 

Therefore, \( \rho = \int f(x) \left[ \beta_1 g_1(x) + \beta_2 g_2(x) \right] \, dx \) 

If the coefficients \( \beta_1 \) and \( \beta_2 \) can be determined, the estimate \( \rho \) of \( \rho_p \) at each pixel would be given by

\[
\rho = \beta_1 \rho_1 + \beta_2 \rho_2
\]  

By comparing eqs. (2) and (5) it is clear that we have to approximate the spectral curve \( g_\rho(x) \) by the linear combination \( \beta_1 g_1(x) + \beta_2 g_2(x) \). A reasonable criterion to adopt is based on the least squares procedure, that is, we want to minimize

\[
\left[ g_\rho(x) - \left( \alpha_1 g_1(x) + \alpha_2 g_2(x) \right) \right]^2 \, dx
\]  

by the appropriate choice of \( \alpha_1 \) and \( \alpha_2 \). In order to do this we discretize the curves \( g_\rho(x), g_i(x) \)
I. For reasons that will be explained later, we consider the case where the rank of the matrix $G$ is given by the number of columns (2 in this case). The least squares solution of this overdetermined system is given by

$$
\hat{a} = (G^T G)^{-1} G^T r
$$

(8)

where $G$ is the Moore-Penrose pseudoinverse of $G$ (Lewis and Odell, 1971).

For reasons that will become clear in the next section, one may want to impose an additional constraint on the values of $a_1$ and $a_2$, namely that $a_1 + a_2 = 1$. In matrix form, this can be expressed as

$$
A a = t, \quad \text{where } A = \begin{bmatrix} 1 & -1 \end{bmatrix} \text{ and } t = 1
$$

(12)

This linear equality constrained least squares problem has the solution given by (Lewis and Odell, 1971):

$$
\hat{a} = \hat{a} + (G^T G)^{-1} G^T (t - A \hat{a})
$$

(13)

where $\hat{a}$ is given by equation (11).

3. SIGNAL TO NOISE RATIO CALCULATION

Let us assume that the linear constraint given by eq. (12) is imposed. In practice, it was verified that the components $a_i$ and $s_i$ are positive so no inequality constraints have to be imposed.

In order to calculate the signal to noise ratio of the simulated band and compare with the signal to noise ratio of the bands $b_1$ and $b_2$, let us assume that each of these bands is degraded by additive noise, i.e.

$$
b_1 = s_1 + n_1 \quad b_2 = s_2 + n_2
$$

(14)

Assume for simplicity (without any loss of generality) the following conditions:

$$
E(s_1) = E(s_2) = 0
$$

(15)

$$
E(n_1^2) = E(n_2^2) = \sigma^2
$$

(16)

Furthermore, let us also assume uncorrelated noise processes on the two bands, i.e.

$$
E(n_1 n_2) = 0
$$

(18)

For each multispectral band

$$
\text{SNR}_b = \frac{s_b^2}{\sigma^2}
$$

(19)

For the simulated band given by

$$
\hat{b} = \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \hat{n}_1 \hat{a}_1 + \hat{n}_2 \hat{a}_2
$$

(20)

we have

$$
\text{SNR}_\hat{b} = \frac{\hat{s}_1^2}{\hat{n}_1^2} + \frac{\hat{s}_2^2}{\hat{n}_2^2}
$$

(21)

We need an additional assumption which is frequently verified in practice, namely, high correlation between multispectral bands, i.e.

$$
P_{\hat{s}_1 \hat{s}_2} \approx 1
$$

(22)

Then, it can easily be shown that

$$
\text{SNR}_\hat{b} \approx \frac{\hat{s}_1^2}{\hat{n}_1^2}
$$

(23)

But $1 - 2P_{\hat{s}_1 \hat{s}_2} \leq 1$ since $\hat{s}_1$ and $\hat{s}_2$ are correlated. Therefore

$$
\text{SNR}_\hat{b} > \text{SNR}_b
$$

(24)

This result shows that an improved signal to noise ratio is obtained in the simulated band, let us remark that the key assumptions leading to this derivation are: 1) linear constrained estimators ($\hat{a}_1 + \hat{a}_2 = 1$) and 2) high correlation between the combined bands $b_1$ and $b_2$. It should be observed that in the experimental results we obtained a correlation coefficient of .89 between $b_1$ and $b_2$.

4. SIMULATION USING SPOT IMAGES

The use of only XS1 and XS2 SPOT multispectral bands to simulate a spatially degraded SPOT panchromatic band is justified by the fact that XS3 has little overlap with the P-band (see Figure 1).

Figure 1 - Typical Spectral Sensitivity of HVR Instruments (source: CNES; SPOT-Image, 1986.}

The spectral curves of XS1, XS2 and P were discretized with 10 points (i.e., n=10), leading to the following numerical results for the unconstrained solution.
Figure 2 shows the discretized XS1 and XS2 curves. Figure 3 displays the discretized P curve, the unconstrained estimated P curve and the linear constrained estimated P curve.

5. EXPERIMENTAL RESULTS
5.1. Spatial Simulation of 20m - Resolution SPOT Panchromatic Band
Since bands XS1 and XS2 have a sampling rate of 20m, we degraded the 10m P band, according to the following two steps.

a) Step 1: Spatial Filtering of the 10m Resolution Panchromatic Band
The P-band was spatially degraded by applying twice the low pass linear filter with finite impulse response given by

\[
\begin{bmatrix}
1 & 169 & 337 & 169 \\
0 & 1 & 412 & 826 & 412 \\
0 & 0 & 1 & 3000 & 169 & 337 & 169 \\
\end{bmatrix}
\]

This impulse response was obtained following the procedure suggested by (Banon, 1990) and considering the attenuation factor \( \gamma \) of the modulation transfer function at half the sampling rate (that is, at Nyquist frequency) given in Table 1.

<table>
<thead>
<tr>
<th>Band</th>
<th>P</th>
<th>XS</th>
</tr>
</thead>
<tbody>
<tr>
<td>row</td>
<td>0.27</td>
<td>0.21</td>
</tr>
<tr>
<td>column</td>
<td>0.16</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 1 - Attenuation Factor \( \gamma \)

The value for P-band are those of (CNES; SPOT Image, 1986); the values for XS2-band are not those of the previous reference but have been reevaluated so that the visual impression of the simulated panchromatic band be similar to the multispectral bands from the point of view of spatial resolution.

b) Step 2: Resampling of the Filtered 10m Resolution Panchromatic Band
The filtered 10m resolution panchromatic band of step 1 was resampled at 0.5 spatial rate, ie, keeping only the first pixel of each pair of consecutive pixels and keeping only the first line of each pair of consecutive lines. Figure 4 shows the result of the spatial simulation of the 20m resolution panchromatic band.

It should be noted that the filtering and resampling procedures could also be done using multirate digital signal processing techniques, according to (Fonseca; Mascarenhas, 1988).

5.2. Spectral Simulation of 20m - Resolution Panchromatic Band
5.2.1. Unconstrained Simulation
The spectrally simulated image has lower mean and lower variance than the spatially simulated band. The reason for this lies in the fact that the spectral curves do not incorporate offset and gain factors that are used in ground processing.

5.2.2. Constrained Simulation
It was observed that the histogram of the spectrally simulated image under a linear constraint is closer to the histogram of the spatially simulated image than the previous case; however, mean and variance of spectrally simulated image are still lower (although closer) than the spatially simulated image.

5.2.3. Gain and Offset Adjustment
The adopted procedure is to introduce the gain and offset adjustments so that the mean value and the standard deviation of the estimated band (either through the unconstrained or the constrained estimation) are equal to those in the unconstrained case: \( P = \alpha_0 P_0 + b_0 \) such that \( E (P') = E (P_0) = (p_0) \) where \( P' \) is the spatially simulated panchromatic band. Analogous expressions are valid for the constrained estimators. In order to avoid increasing round off errors \( \alpha_0 \) and \( \omega_0 \) where combined by applying the transformation directly on \( b_0 \) and \( b_0 \). It was verified that the histogram of the gain and offset compensated estimator without constraint approximates better the histogram of the spatially degraded panchromatic band than the constrained estimator.

5.2.4. Mean Square Error Between Images
The m.s.e. between the spatially simulated band (\( P_0 \)) and the spectrally simulated bands without constraint (\( P_0 \)), with constraint (\( P_0 \)), without constraint with gain and offset compensation (\( P_0 \)) and with constraint with gain of offset...
compensation ($P_{e}$) were computed, according to the expression

$$d = \frac{1}{256 \times 256} \sum_{i=1}^{256} \sum_{j=1}^{256} \left( (X(i,j) - Y(i,j))^2 \right)^{1/2}$$

these results are displayed on Table 2.

<table>
<thead>
<tr>
<th>$P_{e}$</th>
<th>$P_{e}$</th>
<th>$P_{e}$</th>
<th>$P_{e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.75</td>
<td>4.87</td>
<td>2.65</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Table 2 - Mean Square Error Between Spatially and Spectrally Simulated Bands

One can observe that: a) $P_{e}$ approximates $P_{e}$ better than $P_{e}$; b) the results for $P_{e}$ and $P_{e}$ are approximately the same. It should be observed that the comparison between the spectrally simulated band and the spatially degraded band required a small translational registration procedure that was performed manually. It is expected that even smaller mean square errors could be obtained if no registration were necessary.

5.2.5. Visual results

Figure 4 displays 256 x 256 images of $P_{e}$, $P_{e}$ and $P_{e}$. Figure 5 displays $P_{e}$, $P_{e}$, $P_{e}$ and the absolute value of the difference image ($P_{e} - P_{e}$), through a linear contrast stretch that increased the maximum value of the absolute value of the difference image from approximately 16 to 85 for visual purposes. Figure 6 displays the SPOT multispectral bands X51(b1) and X52(b2).

It is observed that $P_{e}$ has lower contrast than $P_{e}$ due to unknown gain and offset factors. $P_{e}$ approximates $P_{e}$ better than $P_{e}$, which is in accordance with Table 2. Furthermore, $P_{e}$ and $P_{e}$ are very close to $P_{e}$ in visual terms.

6. CONCLUSIONS

Through least squares criteria the possibility of simulating one band by linear combination of bands that spectrally overlap the desired band has been demonstrated. Numerically and visually acceptable results were obtained.

BIBLIOGRAPHICAL REFERENCES


